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# Amplitude-domain mechanical spectroscopy: a way to systematic analysis

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**Abstract**

While mechanical spectroscopy analysis has been carried out mostly in the frequency domain (especially via temperature), the amplitude domain has till now been far less considered. Some possible reasons for the above are indicated and amplitude-domain mechanical spectroscopy (ADMS), a promising investigation tool in materials science and engineering, is briefly introduced.

Concerned mainly with elastic and dissipative phenomena, the amplitude-domain mechanical spectra are related to some singularities or peculiarities in the so-called internal friction (or modulus) amplitude-dependent curves (ADCs). A phenomenological analysis is thus proposed, which proved useful for translating the ADCs into the relevant damping spectra *vs.* amplitude, thus performing ADMS.

The observed repetitiousness of likely fractalic regularities observed in ADCs seems to encourage a general model. A possible guideline in such a synthesis effort is indicated as an extension, in terms of generalized mobile entities and pinning agents, of a previously advanced dislocation damping model which uses the device of modelling the new complex picture mostly by a stretched–multipinned–twanged string.

## 1. General background

Most of the work on internal friction (IF) and dynamic modulus is carried out in the temperature domain and such an approach, though somewhat impure, essentially represents a frequency-domain analysis. The reasons for such a prevalence can be summarized as follows: (i) the peaks are generally quite evident in the temperature-domain curves; (ii) the theoretical background is fairly well established; (iii) the relaxation times and activation energies or volumes are easily obtainable; (iv) in most cases, the experimental difficulties are minor.

The major interest in developing amplitude-domain investigations can be justified as follows: being directly expressed in terms of stresses and strains, the amplitude-domain analysis represents a quite immediate information source, especially for structural materials, whose main property is simply the mechanical strength.

Connected with elastic and dissipative phenomena, amplitude-domain mechanical spectroscopy (ADMS) is based on IF (or modulus) amplitude-dependent curves (ADCs). Though the ADCs have been considered by various researchers of the IF family, it can hardly be said that ADMS represents an established and widely used technique.

Apart from most people's reluctance to face nonlinear problems, the main reason for this can be seen in the general tendency to disregard any singularity in the ADCs and to consider any unexpected or strange trend of them as an experimental error rather than the expression of a physical effect. Till today, any peaks in ADCs have mostly been considered as "anomalous".

An extended review of experimental data, devoted to attaining a general phenomenological picture of the ADCs shape and trend, has previously been carried out by one of the present authors [1–5]. For any details we must refer to the quoted preceding papers.

The main conclusion, from the phenomenological point of view, was that ADCs tend to be typically discontinuous, being composed of one or more sequences of linear, parabolic and sigmoidal (LPS) segments (LPS sequences).

In the present paper, we want to point out three more points: (i) the fact that a parametric analysis derivable from the ADCs discontinuous trend represents a self-consistent way to perform ADMS; (ii) the rising view that ADCs are fractalic in nature, in the sense that their "cumulative" trend can reproduce the trend of one or more of their linear, parabolic or sigmoidal component segments; (iii) the guidelines we are following in order to pursue an amplitude-dependent damping model congruous with the above-mentioned discontinuous trend and repetitive phenomenology.


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## 2. The "parametric analysis" of ADCs as a tool for ADMS – assumed fractalic nature of ADCs

The assertion about the LPS nature of the trend in ADCs means, with reference to Fig. 1, that each component segment of an ADC is describable by one of the following forms: for the linear case

$$\Delta_{H0,i} = a(\epsilon - \epsilon_{0,i}) \quad (1)$$

For the parabolic case

$$\Delta_{H1,i} = \beta_i(\epsilon - \epsilon_{1,i})^{\alpha_i} \quad (2)$$

and for the sigmoidal case

$$\Delta_{H2,i} = B_i \exp[-A_i/(\epsilon - \epsilon_{2,i})] \quad (3)$$

where $\epsilon$ is the maximum amplitude of the alternating strain, $i$ is an index represented by one letter of the sequence m, n, p, q, r, s, t (depending on the "order" of the LPS sequence considered) and all the other quantities and parameters are intrinsically positive (or at least null). As to their meaning, the following considerations apply (see also Fig. 1). The quantities in the first terms of eqns. (1)–(3) can be considered as (purely) amplitude-dependent (*e.g.* hysteretic) components of the IF. The quantities $\Delta_{I0,i}$, $\Delta_{I1,i}$ and $\Delta_{I2,i}$ in Fig. 1 can be considered as additive amplitude-independent (*e.g.* dynamic) IF components (to be added to the relevant amplitude-dependent components, in order to obtain the total damping). The quantities $\epsilon_{0,i}$, $\epsilon_{1,i}$ and $\epsilon_{2,i}$ represent the critical strain amplitudes at which trend discontinuities take place and the IF components with the same indexes arise. The remaining factors such as $a_i$, $\alpha_i$, $\beta_i$, $A_i$ and $B_i$ are parameters equal to and very precisely defining the shape of each component segment of the ADC considered.

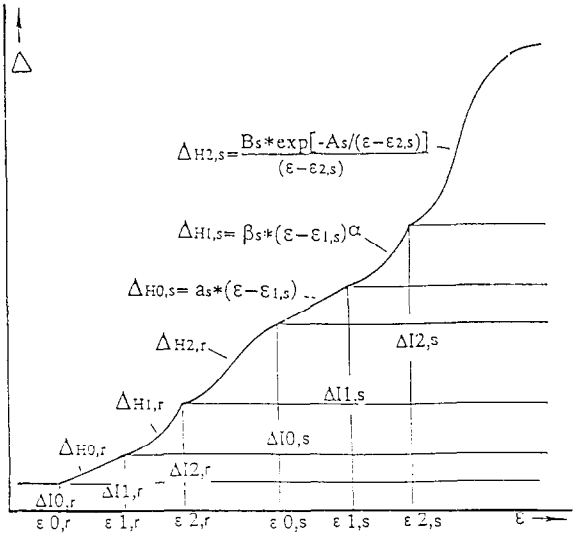


Fig. 1. Schematic representation of an (IF) ADC characterized by two consecutive sequences of LPS segments. Δ represents the total damping (log decrement) and the indices H and I denote amplitude-dependent and amplitude-independent components respectively. $\epsilon$ is the (maximum) amplitude of the alternating strain and $\epsilon_{0,i}$, $\epsilon_{1,i}$ and $\epsilon_{2,i}$ represent critical transition values.

Given an ADC expressed by a set of experimental points, we designate as "parametric analysis" any procedure allowing us to determine the pertinent values of the parameters and components appearing in eqns. (1)–(3) and Fig. 1.

The parametric analysis appears to be of interest, in principle, for the two following reasons: (i) it allows us to express the ADCs shape and trend by a defined set of numbers, thus changing the ADC shape from a qualitative to a quantitative monitor of the material's properties; (ii) it splits the ADC into its various IF components and exactly locates, in the amplitude scale, where each of them come; thus it represents a self-consistent tool to perform ADMS, as anticipated in the preceding section.

As an example, let us consider a computer-aided parametric analysis we carried out on some amplitude-dependent IF results obtained by Marenco and Povolo on oxygen-doped niobium (see ref. 6, Fig. 7).

The analysis results are reported here in Fig. 2, where the values of the independent components, critical amplitudes and parameters obtained for one of the curves (121 K) appear in the table, as an example. The full curves represent the segments (calculated following eqns. (1)–(3)), which contribute to make up each interpolating (discontinuous) curve.

The following purely phenomenological considerations apply.

(a) Though less pronounced than in many other cases the LPS fine structure is nevertheless easily identified.

(b) The fit between the experimental points and the calculated curves is good (standard deviation always lower than 0.002) and much better than with any

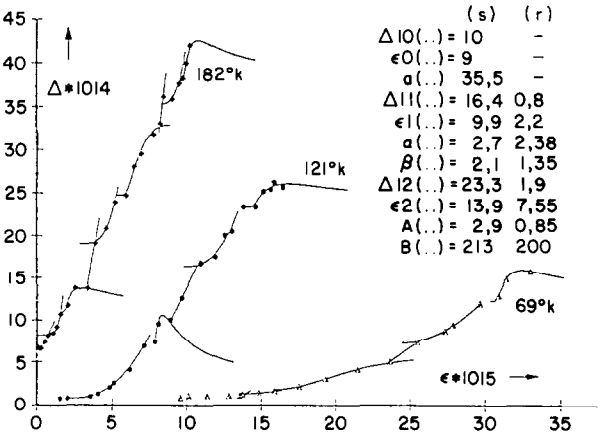


Fig. 2. An example of ADMS parametric analysis as applied to experimental points (log decrement Δ *vs.* amplitude $\epsilon$) obtained by Marenco and Povolo (see ref. 6, Fig. 7).

continuous smooth curve. For instance, the best fit in Fig. 7 of the original paper by Marenco and Povolo shows a standard deviation higher than 0.02; however, various attempts made by us to interpolate the points of Fig. 2 with smooth curves did not attain a standard deviation value lower than 0.008.

(c) Two LPS sequences are present in the 69 and 121 K curves, while some points at the lowest (strain) amplitude could suggest the possibility of a third sequence; and such a low amplitude $q$ sequence is easily detectable in the 182 K curve.

(d) By comparing the curves pertaining to the different temperatures, it can be seen that each component present in a curve can find its counterpart in another curve.

(e) The "gross trend" of the ADCs in Fig. 2 resembles a linear (182 K curve), a sigmoidal (121 K curve) or a parabolic (69 K curve) trend; this means that their gross trend too can resemble one of the typical trends of their component segments; and these are typical features of fractal behaviour.

All these statements, in confirming the repetitious-composite nature of ADCs, seem to encourage: the parametric analysis as a systematic approach to ADMS; the hypothesis of the fractalic nature of ADCs; the pursuit of a generalized model for amplitude-dependent damping.

## 3. Physical–structural interpretation and model

In the preceding section, the capability of the parametric analysis as a self-consistent tool for splitting an ADC into its components, thus performing ADMS, has been illustrated in merely phenomenological terms. However, in order to make ADMS a materials science investigation tool, as far as possible self-consistent, a physical interpretation and model are needed. Given that the phenomenology found is anything but simple and that it deals with non-linear problems, we cannot pretend to be rigorous and exhaustive here in our pursuit of such a goal. We will only try to delineate some possible guidelines.

For this purpose, it may be useful go back to a qualitative model previously advanced for dislocation damping (see ref. 1, plus some additions from refs. 2–5). The previous picture is a variant of the Granato–Lücke (GL) model [7]; it represents an attempt to justify the possible existence of the linear and parabolic IF components, in the low-amplitude side of the curves. This existence, not provided by the GL theory, was ascribed to the possibility that some of the potential pinning atoms would form a somewhat extended impurity cloud around the dislocation, instead of strictly self-locating on its core (as the GL theory considers). In such hypothesis, the linear ($\Delta_{H0}$) and the parabolic ($\Delta_{H1}$) components of the damping were ascribed to the movement of the dislocation inside its impurity cloud. Namely, the two lines of behaviour (linear and parabolic) were related to two different ways of dislocation motion inside the cloud. It had been supposed [2] that movement as in Fig. 3(a) could take place at the lowest amplitudes, just beyond a critical value $\epsilon_0$, with a configuration mainly controlled by the dislocation flexibility. The linear increase of the damping had been ascribed to this kind of motion.

At higher amplitudes (greater than $\epsilon_1$), when the controlling factor becomes the line tension of the dislocation, a movement as in Fig. 3(b) could take place, with a parabolic increase of the damping.

At even higher amplitudes, above a critical strain amplitude $\epsilon_2$, the breakaway phenomena would take place as provided by the GL theory.

Consequently, the possibility of applying the GL plot [7] to the sigmoidal (GL) component $\Delta_{H2}$ of the damping was considered; it was assumed that the $L_c$ value [7] thus obtained would continue to represent the mean interpinner distance (along the dislocation path) at the distance from the dislocation core at which the breakaway process takes place. This can represent very useful quantitative data, irrespective of the fact that a quantitative relationship with microstructure is not available for $\Delta_{H0}$ and $\Delta_{H1}$.

This is, briefly, the substance of the previous model; for any more details we refer to the previous papers [1–6].

At the time the previous model [1] was advanced, the fact that more than one LPS sequence may be

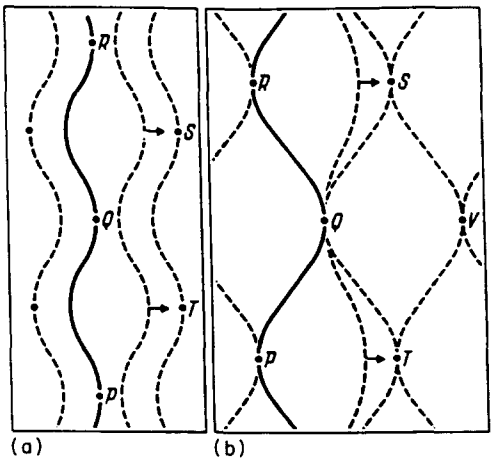


Fig. 3. Two alternatives concerning the motion of a dislocation inside an extended pinning atmosphere: (a) configuration and kind of movement controlled by the dislocation's flexibility; (b) configuration and kind of movement controlled by the dislocation's line tension.

found in an ADC and the consequent major complexity of ADCs were not yet known. So the previous model, like the GL theory, dealt with dislocations only.

The subsequent observation of a fully analogous phenomenology, in materials and at amplitudes where it is hard to think that pure dislocation damping be operative, induces us to consider a further extension.

This extension, a generalized vibrating-string model, assumes that many other structural defects (and defect compositions) can move and dissipate energy nearly as dislocations; or, if preferred, as pinned strings, vibrating in a dissipative medium.

In this context the more or less complex "defects" that could play a role in the amplitude-dependent damping should be classified into two general classes: mobile entities (or moties, for short) and pinning agents (or pinners for short).

Into the first class (moties), microstructural entities are allocated whose movement could give rise in some way to internal damping. Some examples of moties could be represented by: single dislocations, dislocation structures, stacking faults, subgrain boundaries, grain boundaries, magnetic domain boundaries, magnetic flux lines, polymeric chains, embedded fibres in composites etc. (point defects are not considered).

Into the second class (pinners), microstructural entities are allocated which may be able to block or limit in some way the motion, inside the matrix, of the moties. Some examples of these kind of obstacles could be represented by: solute atoms, vacancies, jogs, dislocation intercepts, solute atom aggregates like Guinier Preston zones, heterogeneous components such as precipitates or dispersed phases, interceptions between embedded fibres or polymeric chains etc. Among the possible moties, entities which are not just line defects have also been included; similarly, entities which look much more complex than point defects have been included among the possible pinners. Having established the above general classification, the following considerations or assumptions are made.

(a) The various kinds of moties are more or less mobile depending on both their intrinsic mobility and the pinning effectiveness of the interacting pinners. Thus the whole disposable amplitude range can be divided into subranges where only one kind of motie–pinner couple represents the critical (or even the main) controlling factor.

(b) It is expected that, in the subrange of index $i$, the pertinent moties have the possibility of becoming hysteretically mobile, provided a given critical stress amplitude $\epsilon_{0,i}$ is reached. Their motion would be able to contribute to amplitude-dependent damping through a linear component $\Delta_{H0,i}$ and/or a parabolic component $\Delta_{H1,i}$ until a breakaway phenomenon started taking place. In this case, the pertinent moties would be pulled away from the pinners that were critical at the amplitude considered, thus giving rise to a sigmoidal damping component $\Delta_{H2,i}$.

(c) The assumption is made that any presence of a sigmoidal component of damping in the ADC represents a symptom of a breakaway phenomenon taking place at the corresponding strain amplitude. this means that the moties belonging to the considered subrange start to abandon their current (critical) pinning obstacles to begin a longer path in the matrix.

In this picture it becomes nearly automatic to assume that the critical motie–pinner couples should be responsible for the amplitude-dependent component pertaining to the amplitude subrange considered.

The origin of the amplitude-independent components such as $\Delta_{I0,i}$, $\Delta_{I1,i}$ and $\Delta_{I2,i}$ is, conversely, less clear. We hypothesize in this picture that free and well-anchored moties may be responsible for these components by some dynamic mechanism. Namely, the well-anchored moties could contribute by a mechanism analogous to the GL dynamic mechanism [7]. The free moties could additionally contribute by a presently unknown mechanism, *e.g.* by a twanging effect due to the pinners, acting as the tooth of a bidimensional comb plectrum.

## 4. Discussion and conclusions

Owing to the non-linear character and the relevant phenomenological complexity of amplitude-dependent damping, the picture outlined at the end of the previous section cannot rise, at present, to the rank of a fully quantitative model. It should mostly be regarded as a frame of a mosaic into which to locate the pieces of theory proposed from time to time.

An already asserted element of such a mosaic is represented by the GL equation (eqn. 2.1 of ref. 7, p. 790), expressing the amplitude-dependent dislocation damping. In the present view, a very general validity of that equation is postulated. It is assumed it can be valid, *mutatis mutandis*, for most couples of moties and pinners and not only for dislocations and dilute pinning atoms (at very low temperatures). In this sense we affirmed that useful information on the interpinner distance at breakaway can be drawn from the GL plot.

Another example may be represented by eqn. 2.2 of the same GL paper, which may be able to yield information presumably connected with damping contributions (*e.g.* the $\Delta_{I0,i}$ components) due to the still anchored moties.

The proposed ADMS systematic approach (the parametric analysis) is able to: (i) yield the parameters defining, via GL formulas, the hysteretic and some dynamic components of the IF yielding *inter alia* data

on the interpinner distances at the various breakaways; and (ii) detect all the critical amplitudes $\epsilon_{0,i}$, $\epsilon_{1,i}$ and $\epsilon_{2,i}$ at which "something new happens" concerning micromechanical behaviour. Hence the proposed picture changes from a purely qualitative to a partly quantitative model.

Other remarks must be deferred. Only one more consideration should be allowed. Sigmoids (see also Fig. 2) by their nature present a maximum: the possibility that the so-called "anomalous peaks" sometimes found in ADCs may be simply related to such maxima, *i.e.* to the breakaway phenomena, should be at least considered.

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